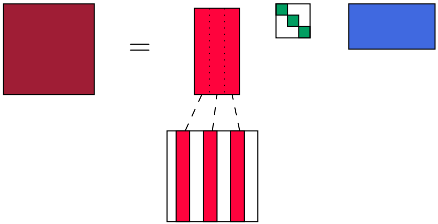
Démélange spectral d'images hyperspectrales en présence de pixel purs.



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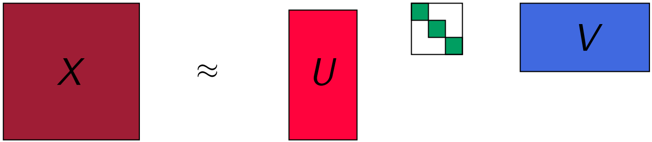
Nonnegative Matrix Factorization



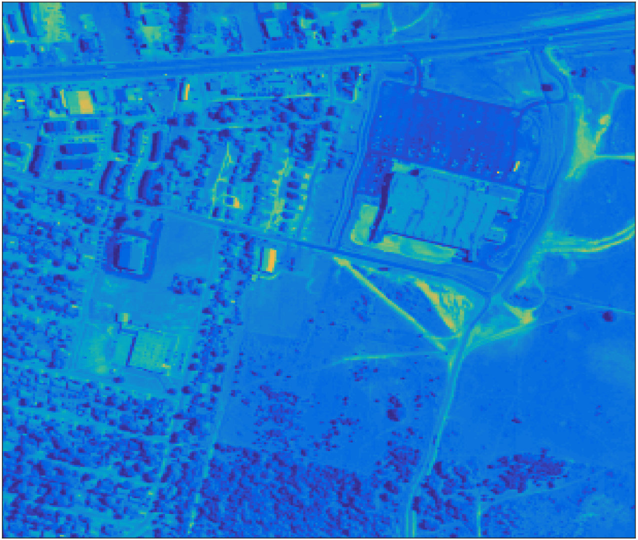
$$X \approx UV \iff X(:,j) \approx \sum_{k=1}^{k} U(:,k)V(k,j)\ \forall j.$$

where $U \geq 0$ and $V \geq 0$ element-wise.

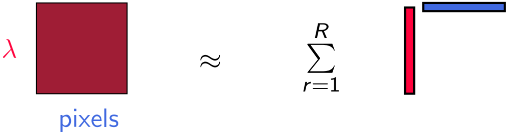
Uniqueness

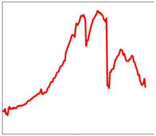
The NMF of a nonnegative matrix X is **not unique** unless some harsh sparsity conditions on X are met [Donoho 2004, Laurberg 2008, Huang 2013].

Application to spectral unmixing of HSIs

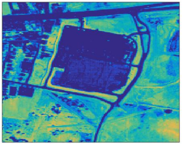


Application to spectral unmixing of HSIs





Spectre(λ)



Répartition spatiale(pixel)

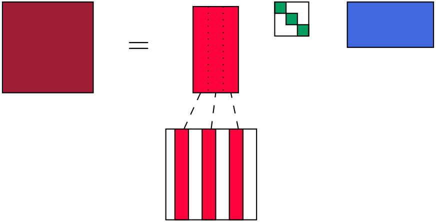
$$X \approx \sum_{r=1}^{R} \mathbf{u}_r \mathbf{v}_r \approx UV$$

Challenges

Identification problem : Which materials are present in the scene ?

Unmixing problem : What is the composition of each pixel ?

Additional hypothesis : Separability



$$X = UV \text{ where } U = X(:,\mathcal{K}) = XS$$

- Same formalism can be used if an external library is available.

Uniqueness

DNMF is **unique** if spark$(D) \geq R$ where R is the rank of X. [C. 2017]

State-of-the-art (non-exhaustive)

- **Continuous approaches ($X = DSV = DY$)**
  Lasso, GLUP [Ammanouil 2014], FGNSR [Gillis 2016]
  + Robust, optimization criterion. − Slow.
- **Greedy/Non-iterative method ($X = D(:, K)V$)**
  - Geometric algorithms (pure pixel hypothesis)
    N-FINDR [Winter 1999], VCA [Nascimento 2005], SPA [Gillis 2014, Businger Golub 1965]
  - Matching pursuit approaches
    SDSOMP[X.Fu 2013, Tropp 2006], MPALS [C. 2017]

  + Fast − No explicit optimization criterion
- **Pixel-wise brute force algorithms**
  MESMA [Roberts 1998], MESLUM, AUTOMCU, AMUSES [Degerickx 2017]
  + Flexible − No low rank property, Slow.
- **Statistical methods**

VCA (Vertex Component Analysis) [2005]

- Compute a random direction, orthogonal to previously selected spectra.
- Find the largest spectra along this direction and add to the set of endmembers.
- Repeat R times.

✓ Fast

✗ Sensitive to Noise

✗ Non-deterministic

NFINDR [1999]

- Take a set of current estimates of endmembers, and a random spectrum.
- Replace each current estimate with the picked spectrum, and check is volume increases.
- Repeat until convergence.

✓ Explores a large set of configurations.

✗ Sensitive to Noise

✗ Slow for large data set or large number of endmembers

Successive Projection Algorithm (SPA) [2014]

- Choose the spectrum with largest norm.
- Orthogonalized the remaining spectra with respect to all previously selected endmembers.
- Repeat R times.

✓ Very fast

✓ Robust to small noise

✗ Not applicable in multispectral imaging

✗ May output poor reconstruction error in practice

Matching Pursuit Alternating Least Squares (MPALS) [2017]

- Find best spectra (not in data) knowing the abundances
- Find normalized endmembers in data as close as possible to previously computed spectra.
- Find abundances knowing endmembers
- Repeat until convergence

✓ Adaptable to various scenario (tensor data, near-separable data...)

✓ Accounts for spectral variability

✓ Provides small reconstruction error

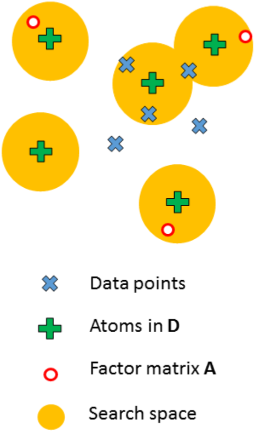
✗ No convergence proof

✗ Sensitive to initialization (use other methods like SPA to initialize)

1. Introduction on Separable NMF
2. Some algorithms for separable NMF
3. Perspectives

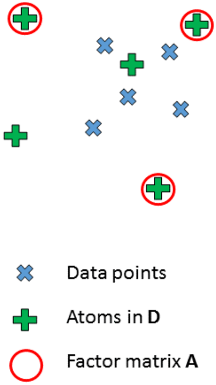
Towards flexibility in the pure-pixel assumption

Flexibility



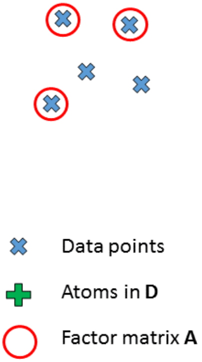
$$\mathbf{A} \approx \mathbf{DS}$$

Standard case



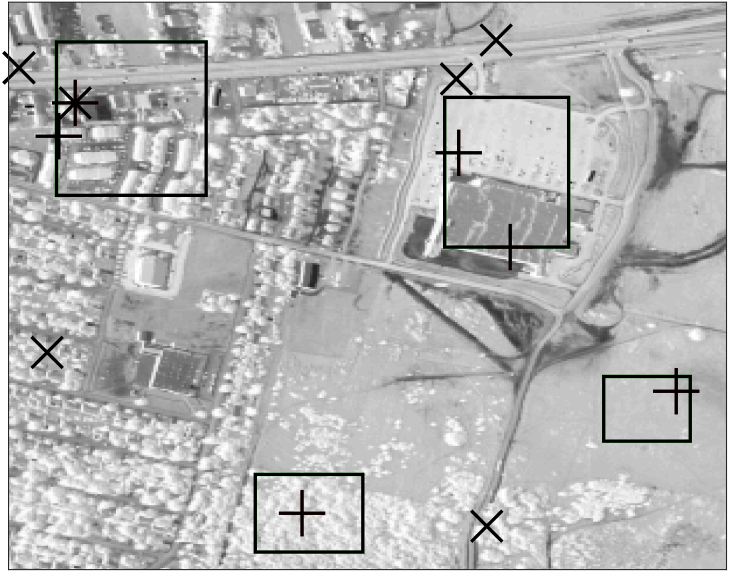
$$\mathbf{A} = \mathbf{DS}$$

Separability

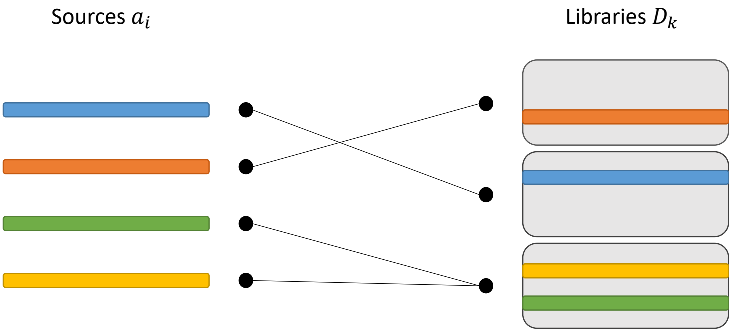


$$\mathbf{A} = \mathbf{XS}$$

Pure pixel automatic selection in a zone



Multiple Dictionaries



$$\mathbf{A} = [\mathbf{D}_1(:, \mathcal{K}_1), \ldots, \mathbf{D}_p(:, \mathcal{K}_p)]\mathbf{\Pi}$$

Thank you for your attention !